FORCED VIBRATIONS:— B.Sc-I, Paper-I, Group-B.

The time period of a body executing simple harmonic motion depends on the dimensions of the body and its elastic properties. The vibrations of such a body die out with time due to dissipation of energy. If some external periodic force is constantly applied on the body, it continues to oscillate under the influence of such external forces. Such vibrations of the body are called forced vibrations.

Initially, the amplitude of the swing increases, then decreases with time, becomes minimum and again increases. This will be repeated if the external periodic ~~ext~~ force is constantly applied on the system. In such case the body will finally be forced to vibrate with the same frequency as that of the applied force. The frequency of the forced vibration is different from the natural frequency of vibration of the body. The amplitude of the forced vibration of the body depends on the difference between the Natural frequency and the frequency of the applied force. The amplitude will be large if difference in frequencies is small and vice versa.

For forced vibrations, ~~eqn~~ are (or damped.)

$$m\frac{d^2y}{dt^2} + ky + \mu\frac{dy}{dt} = 0$$

This eqn modified in the form is

$$m\frac{d^2y}{dt^2} + ky + \mu\frac{dy}{dt} = F \sin pt \quad \text{—(1)}$$

let the external periodic force be represented by $F \sin pt$ where F is the maximum value of the force of frequency $\frac{p}{2\pi}$.

Here P is the angular frequency of the applied peak force.

The particular solution of eqn ① representing the forced vibrations is

$$y = a\sin(pt-\alpha) \quad \text{—②}$$

$$\therefore \frac{dy}{dt} = pa\cos(pt-\alpha) \quad \text{—③}$$

$$\frac{d^2y}{dt^2} = -p^2 a\sin(pt-\alpha)$$

$$\text{So } \frac{d^2y}{dt^2} = -p^2 y \quad \text{—④}$$

Substituting these values in eqn ①

$$-mp^2 a\sin(pt-\alpha) + ka\sin(pt-\alpha) + \mu ap\cos(pt-\alpha) = F\sin pt$$

$$= -mp^2 a[\sin pt\cos\alpha - \cos pt\cdot\sin\alpha] + ka[\sin pt\cdot\cos\alpha - \cos pt\cdot\sin\alpha] + \mu ap[\cos pt\cdot\cos\alpha + \sin pt\cdot\sin\alpha] - F\sin pt = 0 \rightarrow ⑤$$

when $\sin pt = 1$, $\cos pt = 0$

$$\therefore -mp^2 a\cos\alpha + ka\cos\alpha + \mu ap\sin\alpha - F = 0 \quad \text{—⑥}$$

when $\cos pt = 1$, $\sin pt = 0$

$$\therefore mp^2 a\sin\alpha - ka\sin\alpha + \mu ap\cos\alpha = 0 \rightarrow ⑦$$

dividing eqn ⑦ by $\cos\alpha$ and simplifying

$$\tan\alpha = \frac{\mu p}{k - mp^2} = \frac{A}{B} \quad \text{—⑧}$$

from eqn ⑧ $\quad \sin\alpha = \frac{A}{\sqrt{A^2+B^2}} \quad$ —⑨

$$\cos\alpha = \frac{B}{\sqrt{A^2+B^2}} \quad \text{——} (10)$$

Dividing eqn (6) by $\cos\alpha$

$$-mp^2a + Ka + \mu a p \tan\alpha - \frac{F}{\cos\alpha} = 0.$$

or, $$a\left[(K - mp^2) + \mu p \tan\alpha\right] = \frac{F}{\cos\alpha}$$

But $(K - mp^2) = B.$

and $\mu p = A$

Substituting the values of $\tan\alpha$ and $\cos\alpha$

$$a\left[B + \frac{A^2}{B}\right] = \frac{F\sqrt{A^2+B^2}}{B}$$

$$a = \frac{F}{\sqrt{A^2+B^2}}$$

Substituting the values of A and B.

$$a = \frac{F}{\sqrt{\mu^2p^2 + (K - mp^2)^2}} \quad \text{——} (11)$$

$\therefore \; y = a\sin(pt - \alpha)$

or $$y = \frac{F}{\sqrt{\mu^2p^2 + (K - mp^2)^2}} \sin(pt - \alpha) \quad \text{——} (12)$$

Applying the Boundary Conditions, another solution is obtained when $F = 0$.

This Corresponds to free vibrations. In the case of free vibrations the solution is $y = ae^{-bt}\sin(\omega t - \alpha)$

The general solution will include both the particular solutions for free and forced vibrations

$$y = a e^{-bt} \sin(\omega t - \alpha) + \frac{F}{\sqrt{\mu^2 p^2 + (k - mp^2)^2}} \sin(pt - \alpha) \quad (14)$$

Here $b = \frac{\mu}{2m}$.